Ground vibration due to a high-speed moving harmonic rectangular load on a poroviscoelastic half-space

G. Lefeuve-Mesgouez*, A. Mesgouez

UMR A 1114 Climate, Soil and Environment, Université d’Avignon, Faculté des Sciences, 33 rue Louis Pasteur, F-84000 Avignon, France

RUNNING TITLE: Vibrations induced by high-speed moving loads on poroviscoelastic half-space

* Corresponding author. Tel: +33 (0)490144463. Fax: +33(0)490843815.

E-mail address: gaelle.mesgouez@univ-avignon.fr
Abstract

The transmission of vibrations in the ground, due to a high-speed moving vertical harmonic rectangular load, is investigated theoretically. The problem is three-dimensional and the interior of the ground is modelled as a totally or partially saturated porous viscoelastic half-space, using the complete Biot theory. The solutions in the transformed domain are obtained using a double Fourier transform on the surface spatial variables. A modified hysteretic damping model defined in the wavenumber domain is used, first presented by Lefeuve-Mesgouez et al. (2000). Numerical results for the displacements of the solid and fluid phases, over the surface of the ground and in depth, are presented for loads moving with speeds up to and beyond the Rayleigh wave speed of the medium.

Keywords: Moving load; Wave propagation; Porous media; Biot theory; High speed; Mach cone; $P_2$ wave.

1 Introduction

The problem of determining the response of a soil under the action of moving loads has received considerable attention in the last few decades. Research activities in this area have been motivated by the need to determine the vibratory motion on the ground surface and in depth caused by moving vehicles. Moreover, high-speed trains are becoming increasingly common and freight trains increasingly heavier. Combined with this fact and the observation that the Rayleigh wave speeds are slower in soft soils, we note that the study of moving loads is of great importance for environmental and geotechnical engineering. In fact, some problems of high vibrations induced by moving loads have been observed in Sweden, Madshus and Kaynia (2000), and in the North West of France,
Picoux et al. (2003).

In the framework of moving loads, many authors have worked on grounds modelled as viscoelastic homogeneous or multilayered media. Theodorakopoulos (2003) presents a detailed list of recent references in this area for semi-analytical and numerical models. For this kind of modelisation, current works deal with more accurate models of the track and its interaction with the viscoelastic ground. Thus, more recently, Picoux et al. have developed two-dimensional (2003) and three-dimensional (2005) semi-analytical wave propagation models for the response of the ground surface due to vibrations generated by a railway traffic for low vibration frequencies. The authors take into account the railway track and analyse the influence of the moving load speed. Comparison with experimental data shows that semi-analytical models can provide useful information to better understand the behaviour of track and ground due to wave propagation. Comparisons of either semi-analytical models or numerical models with experimental data are also proposed by Sheng et al. (2003) and Paolucci et al. (2003) respectively. Semi-analytical approaches have been developed using Fourier transforms on spatial variable, see for instance Sheng et al. (2004), (2006). Recently, Steenbergen and Metrikine (2007) have presented an overview of models of railway tracks over elastic or viscoelastic half-space. The authors focus on the interface between the beam and the half-space. They determine the domain of validity of three models of interest depending on the load speed, the wavelength compared to the track width and the distance to the track (far or near field). Some results are presented for super-Rayleigh regimes with the visualisation of Mach lines over the surface. Among numerical methods, one can quote Yang et al. (2003) who develop a 2.5 D finite/infinite element approach and present a parametric study of the train induced wave propagation in layered viscoelastic soils: they analyse in particular the influence of the shear wave speed, the damping ratio and the moving load speed.
Finite element models can also be used, see Hall (2003) for instance.

Another area of investigation on loads moving over grounds deals with the model used for the ground. In fact, the soil is composed of a solid skeleton and pore space filled with fluid(s). The Biot theory is widely used to describe the macroscopic two-phase continuum. As for poroviscoelastic models of grounds submitted to moving loads, only a few papers are available. Nevertheless, Theodorakopoulos (2003) shows that in the case of soft materials, models ignoring the coupling between fluid and solid may lead to errors, especially at high velocities. In fact, the importance of the interaction between the fluid component and the solid part of the medium is now generally recognised. Thus, poroelastic models have become of main interest. The author proposes a parametric analysis of a poroelastic half-plane medium under moving loads. The effects of porosity and permeability on the response are more pronounced in soft materials for high load speeds. The theoretical approach is based on Fourier expansion. In the continuation of the previous paper, Theodorakopoulos et al., (2004) and (2006), propose an alternative, approximate method for the analysis of a poroviscoelastic soil medium under moving loads. Nevertheless, these works are restricted to sub-critical load speed cases and two-dimensional geometries. Jin et al. (2004) also consider the two-dimensional dynamic response of a poroelastic half-space, and analyse the stresses. They do not take into account the damping of the solid part and the load speed is still limited to subsonic cases.

The study of two-dimensional models is useful to better understand physical phenomena such as, for example, the superposition of the different waves propagating in front of and/or behind the load. Some general conclusions can be drawn from such studies: Doppler effects, existence of Mach cones in depth, etc. But, two-dimensional models are limited especially when studying the surface of the ground. The study of 2D models allows to have some features of the displacements behind the load but prevents for instance
from the visualization of higher displacement Mach lines due to the Rayleigh wave.

The three-dimensional analytical solution for the dynamic response of a half-space porous medium subjected to a point load applied on the surface and moving with constant speed has been proposed very recently by Lu and Jeng (2007). To the best of our knowledge, it is the only paper available on three-dimensional approach for this kind of subject. The method uses multiple Fourier transforms on time and spatial variables, and the inverse transform is reduced to a double one by the use of Dirac functions. The authors study more specifically the response of three points in the ground and the evolution of their maximum values with increasing load speed. Nevertheless no three-dimensional result and no analysis in the wavenumber domain are presented. In fact, works dealing with loads moving over poroviscoelastic grounds are often restricted either to 2D geometries or to the sub-Rayleigh speed range.

In this paper, the authors propose a three-dimensional semi-analytical approach to study the displacements induced by a more realistic harmonic rectangular load moving at constant speed over the surface of a two-phase poroviscoelastic half-space. The Biot theory including elastic, inertial and viscous couplings is considered. Moreover a modified hysteretic damping is used for the solid phase in order to obtain results for “supersonic” cases, Lefeuve-Mesgouez et al. (2000). Thus, the moving load speed range covers both sub- and super-Rayleigh regimes. Partial differential equations are solved using both Fourier transforms and analytical solution in the wavenumber domain. A preliminary study of dispersion and attenuation is proposed to determine the frequency range for which one equivalent single phase model can be used and the frequency range for which two-phase Biot theory is needed. For this, the characteristic frequency is introduced. Then, the responses are studied in the wavenumber domain which dissociates the body and surface wave contributions and underlines the influence of each wave. Results in the spatial
domain are also presented. Particular attention is paid to the compressional wave of the second kind with the visualisation of Mach cones relative to this $P2$ wave. Moreover, an example of partially saturated porous medium is also studied. For this kind of soil, the $P2$ wave speed can be lower.

Therefore, the article is divided as follows: section 2 presents the governing equations of the problem and the wave equations; section 3 details the used method for the solution of the wave equations and section 4 presents the obtained numerical results for different cases.

2 Theoretical approach

2.1 Vibration transmission

A typical example of the considered model is shown in Fig. 1. A moving harmonic vertical ($x_3$) load acts uniformly over a rectangle whose dimensions are $2l_1 \times 2l_2$. It moves over the surface of a homogeneous, isotropic, poroviscoelastic half-space in the $x_1$-direction at constant speed $c$ with radial frequency $\omega$. The half-space is modelled as a two-phase continuum composed of a porous deformable viscoelastic solid skeleton and a fluid component corresponding to the viscous fluid filling the porous space. The macroscopic equations for dynamic isotropic saturated poroelasticity for small strains in a Lagrangian description were first presented by Biot (1956). Bourbié et al. (1986), or de Boer (2000), have proposed a complete review of the Biot theory. The Biot theoretical approach includes three different couplings between the two phases: elastic, inertial and viscous couplings. In such a medium, three body waves exist: the $P1$ and $P2$ compressional waves and the $S$ shear wave. Moreover, in the case of a semi-infinite medium, a surface wave also exists, denoted as the Rayleigh $R$ wave.
The formulation can be written either with the solid and fluid displacement vectors \( \{u, U\} \) or with the solid and relative displacement vectors \( \{u, w\} \) (with \( w = \phi(U - u) \)). We retain here the second formulation coupled with Helmholtz decompositions on solid and relative displacements: this method leads to simpler symmetrical matrices, see Degrande et al. (1998) or Yang (2000, 2001). Alternative approaches exist: the introduction of dilatations, Jones and Petyt (1991, 1993, 1998), in the restricted case of a viscoelastic one-phase medium, the Helmholtz decompositions of solid and fluid displacements, used for instance by Bourbié et al. (1986), or Al-Khoury et al. (2002).

Then, the equations of the problem can be written as follows:

- the equations of motion

\[
\sigma_{ij,j} = [(1 - \phi)\rho_s + \phi\rho_f]\ddot{u}_i + \rho_f\ddot{w}_i \quad (1)
\]

\[
p_i = -\frac{1}{K}\dot{w}_i - \rho_f\ddot{u}_i - \frac{a\rho_f}{\phi}\dddot{w}_i \quad (2)
\]

- the constitutive relationships

\[
\sigma_{ij} = [\lambda_0 + M\beta^2]u_{k,k}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + \beta Mw_{k,k}\delta_{ij} \quad (3)
\]

\[
p = -M\beta u_{k,k} - Mw_{k,k} \quad (4)
\]

with material properties denoted as \( \lambda_0 \) and \( \mu \) (drained Lamé constants for the purely elastic equivalent porous media), \( \rho_s \) and \( \rho_f \) (solid grains and fluid densities), \( \phi \) (porosity). Moreover, \( \sigma_{ij} \) represent the total Cauchy stress tensor components, \( p \) is the pore pressure in the fluid. \( \delta_{ij} \) is the Kronecker symbol, \( u_i, w_i \) and \( x_i \) are the \( i \)th components of the vectors \( \{u_1, u_2, u_3\} \), \( \{w_1, w_2, w_3\} \) and \( \{x_1, x_2, x_3\} \) respectively, and the summation
convention is applied. The subscripts () and the superscripts (˙) each denote spatial and time derivatives respectively. Eq. (2) corresponds to a generalised Darcy law in transient regimes. Let us now outline the three couplings:

- $K$ is the hydraulic permeability coefficient defined by $K = k/\mu_d$, with $k$ the absolute permeability coefficient and $\mu_d$ the dynamic viscosity of the fluid. It corresponds to the drag forces and thus represents the macroscopic viscous coupling due to the relative motion between the Poiseuille type flow and the solid matrix on the microscopic scale.

- $a$ is the tortuosity factor. It stands for the inertial forces and is obtained from homogeneisation theory. The tortuosity is a macroscopic quantitative measure of how much the pore geometry disrupts the viscous fluid flow at a microscopic scale. In the case of uniform pores, all oriented parallel to the microscopic flow of a perfect fluid and presenting a circular section, note that $a=1$. In the case of an ideal solid matrix made by spherical grains, Berryman and Wang (1980) express the tortuosity factor as $a = (1 + \frac{1}{\phi})/2$.

- $M$ and $\beta$ are the first and second Biot coefficients where $\beta$ represents the macroscopic elastic coupling. Indeed, in Eq. (4), $\beta$ links, for a drained medium ($p = 0$), the solid macroscopic volume modification $u_{i,i}$ to the fluid dilatation $U_{i,i}$. The Biot coefficients are defined as $\beta = 1 - K_0/K_s$ and $1/M = (\beta - \phi)/K_s + \phi/K_f$ where $K_0$, $K_s$, $K_f$ are the bulk moduli of the drained porous medium, the solid grains and the fluid component respectively.

A viscoelastic hysteretic Rayleigh damping is taken into account for the solid phase. For high load speeds, specifically for super-Rayleigh regimes, Lefeuve-Mesgouez et al. (2000) have shown that the standard hysteretic damping model is not suitable for the moving load
problem because it induces some erroneous displacements. Thus they have introduced a modified hysteretic damping model defined in the wavenumber domain as presented in Eq. (5).

\[ \lambda_0 = \lambda_0 S (1 + i \eta \ \text{sign}(\omega - \gamma c)) \quad \text{and} \quad \mu = \mu S (1 + i \eta \ \text{sign}(\omega - \gamma c)) \]  

(5)

instead of \( \lambda_0 = \lambda_0 S (1 + i \eta) \) for the standard hysteretic damping. \( \eta \) is the damping factor and \( \gamma \) the wavenumber issued from a Fourier transform on the spatial variable relative to the moving direction. It can also be written: \( \text{sign}(\omega - \gamma c) = \text{sign}(k - \gamma) \) with \( k = \omega/c \).

Lefeuvre-Mesgouez et al. (2000) have tracked the location of poles and branch points for various load speeds, in the complex plane. For super-Rayleigh cases, with a standard hysteretic damping, a Rayleigh pole gives a contribution in front of the load which is not suitable. With the modified hysteretic damping, this Rayleigh pole jumps to another part of the complex plane and gives a second contribution behind the load as expected. The same remarks can be done for the branch points.

In the case of a homogeneous half-space, the physical parameters do not depend on the spatial variables. Thus, spatially derivating constitutive relationships (3) and (4) and introducing these expressions in Eqs. (1) and (2) yields the following set of vector equations

\[
\begin{aligned}
\left\{ \begin{array}{l}
(\lambda_0 + \mu + M\beta^2)\nabla(\nabla \cdot u) + \mu \nabla^2 u + M\beta \nabla(\nabla \cdot w) = [(1 - \phi)\rho_s + \phi \rho_f] \ddot{u} + \rho_f \ddot{w} \\
M\beta \nabla(\nabla \cdot u) + M \nabla(\nabla \cdot w) = \frac{1}{K} \dot{w} + \rho_f \dot{u} + \frac{\rho_f}{\phi} \dot{w}
\end{array} \right.
\end{aligned}
\]  

(6)

The Helmholtz decompositions on the solid and relative displacements are written as

\[ u = \nabla \varphi + \nabla \times \psi \quad \text{and} \quad w = \nabla \varphi^r + \nabla \times \psi^r \]  

(7)

where \( \varphi \) and \( \varphi^r \) are scalar potentials and \( \psi \) and \( \psi^r \) vector potentials. Then introducing
the following notations

\[
[K_P] = \begin{bmatrix}
\lambda_0 + 2\mu + M\beta^2 & M\beta \\
M\beta & M
\end{bmatrix}
\]

\[
[K_S] = \begin{bmatrix}
\mu & 0 \\
0 & 0
\end{bmatrix}
\]

\[
[M] = \begin{bmatrix}
(1 - \phi)\rho_s + \phi\rho_f & \rho_f \\
\rho_f & \frac{\rho_f}{\phi}
\end{bmatrix}
\]

\[
[C] = \begin{bmatrix}
0 & 0 \\
0 & \frac{1}{\kappa}
\end{bmatrix}
\]

we obtain the two following uncoupled matrix systems, relative respectively to the compressional waves \(P1\) and \(P2\) associated with the Helmholtz scalar potentials \(\varphi\) and \(\varphi^r\), and to the shear wave \(S\) associated with the Helmholtz vector potentials \(\psi\) and \(\psi^r\)

\[
[M]\begin{bmatrix}
\ddot{\varphi} \\
\ddot{\varphi}^r
\end{bmatrix} + [C]\begin{bmatrix}
\dot{\varphi} \\
\dot{\varphi}^r
\end{bmatrix} - [K_P]\begin{bmatrix}
\Delta\varphi \\
\Delta\varphi^r
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

(8)

\[
[M]\begin{bmatrix}
\ddot{\psi} \\
\ddot{\psi}^r
\end{bmatrix} + [C]\begin{bmatrix}
\dot{\psi} \\
\dot{\psi}^r
\end{bmatrix} - [K_S]\begin{bmatrix}
\Delta\psi \\
\Delta\psi^r
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

(9)

2.2 Boundary conditions

A moving harmonic rectangular load is applied over the surface of the half-space \((z = x_3 = 0)\), which leads to the boundary conditions

\[
\sigma_{33}|_{x_3=0} = f(x_1 - ct, x_2)e^{i\omega t}
\]

(10)

\[
\sigma_{13}|_{x_3=0} = \sigma_{23}|_{x_3=0} = p_{x_3=0} = 0
\]

(11)

where \(f(x_1 - ct, x_2)\) is a function depending on the spatial variables \(x = x_1 - ct\) and \(y = x_2\). For a uniformly distributed load over a rectangle with dimensions \(2l_1 \times 2l_2\), \(f\) is
Given by

\[ f(x_1 - ct, x_2) = \frac{f_0}{4l_1l_2} \text{ for } |x_1 - ct| < l_1, \ |x_2| < l_2 \quad (12) \]

2.3 Modelling a partially saturated ground

For a partially saturated poroviscoelastic ground, a third gas phase, some air for instance, has to be taken into account in the description of the medium. Indeed, the connected porosity is filled with both liquid and gas phases. The simplest approach consists in considering that the gas phase amount in the pores is weak so that the gas phase forms small bubbles embedded in the liquid phase. This assumption, thus, leads to a modification of the compressibility of the fluid phase, Verruijt (1969) or Smeulders et al. (1992) for instance, and the unsaturated porous medium can then be described by the classical two-phase solid-fluid Biot theory. The mixture of liquid and gas in the fluid phase is quantified by the degree of saturation \( S_R \) defined by

\[ S_R = \frac{V_{\text{liq}}}{V_{\text{fl}}} \quad (13) \]

where \( V_{\text{liq}} \) and \( V_{\text{fl}} \) respectively represent the volume occupied by the liquid and the volume occupied by the fluid phase, which means the volume of the connected space. The saturated medium presents a degree of saturation \( S_R = 1 \).

By considering that the gas follows the ideal gas law and that it undergoes isothermal transformation, the bulk modulus \( K_{\text{fl}} \) of the homogenised fluid phase is then estimated for a small gas fraction \( (S_R > 90\%) \) by

\[ \frac{1}{K_{\text{fl}}} \simeq \frac{1}{K_{\text{liq}}} + \frac{1 - S_R}{P} \quad (14) \]

Let us note that the presence of a small quantity of gas, even a very small quantity, drastically reduces the bulk modulus of the fluid phase and thus the value of Biot coefficient \( M \). Speeds of the two compressional waves \( P1 \) and \( P2 \) are then modified, the shear
wave $S$ speed is not changed because of its intrinsic independence from the fluid phase, as presented in Tab. 3.

3 Wave propagation solution

3.1 Helmholtz potentials in the wavenumber domain

Introducing the moving coordinate frame, we can write the following change of variables and functions

$$
\varphi(x_1, x_2, x_3, t) = \Phi(x, y, z) e^{i\omega t} \quad \text{and} \quad \varphi^*(x, y, z) e^{i\omega t}
$$

$$
\psi(x_1, x_2, x_3, t) = \Psi(x, y, z) e^{i\omega t} \quad \text{and} \quad \psi^*(x, y, z) e^{i\omega t}
$$

Then, to solve the resulting system, we use a double spatial Fourier transform defined by

$$
\bar{h}(\gamma, \zeta, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y, z) e^{-i(\gamma x + \zeta y)} \, dx \, dy
$$

With the above definition and from Eqs. (8) and (9), matrix differential systems relative to the compressional waves and the shear wave respectively are obtained as follows

$$
\begin{bmatrix}
-\left( \frac{d^2}{dz^2} - \gamma^2 - \zeta^2 \right) [K_P] - (\omega - \gamma c)^2[M] + i(\omega - \gamma c)[C]
\end{bmatrix}
\begin{bmatrix}
\Phi

\Phi^*
\end{bmatrix}
= \begin{bmatrix}
0

0
\end{bmatrix}
$$

$$
\begin{bmatrix}
-\left( \frac{d^2}{dz^2} - \gamma^2 - \zeta^2 \right) [K_S] - (\omega - \gamma c)^2[M] + i(\omega - \gamma c)[C]
\end{bmatrix}
\begin{bmatrix}
\Psi

\Psi^*
\end{bmatrix}
= \begin{bmatrix}
0

0
\end{bmatrix}
$$

Introducing $\alpha_{Pj}$ issued from the following determinant

$$
\det \left[ (\alpha_{Pj}^2 - \gamma^2 - \zeta^2)[K_P] + (\omega - \gamma c)^2[M] - i(\omega - \gamma c)[C] \right] = 0, \quad j = 1, 2
$$

with $j = 1$, for the $P1$ wave and $j = 2$ for the $P2$ wave, the solution of system (18) can
be written as

\[
\left\{ \begin{array}{c}
\Phi^* \\
\Phi^r^*
\end{array} \right\} = \left\{ \begin{array}{c}
\Phi_1^* \\
\Phi_1^r^*
\end{array} \right\} e^{-\alpha P_1 z} + \left\{ \begin{array}{c}
\Phi_2^* \\
\Phi_2^r^*
\end{array} \right\} e^{-\alpha P_2 z}
\]

(21)

Reflections from \( z \rightarrow +\infty \) (downward \( z \)-axis) are disallowed for \( \Re \{\alpha P_j\} > 0 \). Moreover, propagation occurs in the increasing \( z \). Consequently, \( \alpha P_j \) have to be chosen so that \( \Im \{\alpha P_j\} > 0 \) since \( \omega > 0 \).

Furthermore, \( \Phi_1^* \) and \( \Phi_1^r^* \) on one hand and, \( \Phi_2^* \) and \( \Phi_2^r^* \) on the other hand are linked by

\[
\Phi_j^r = \left. \frac{(\alpha^2 P_j - \gamma^2 - \zeta^2)\beta M + (\omega - \gamma c)^2 \rho_f}{(\alpha^2 P_j - \gamma^2 - \zeta^2)M + (\omega - \gamma c)^2 \rho_{f a}/\phi - i(\omega - \gamma c)/K} \Phi_j^* \right) \quad (22)
\]

From Eq. (19), it is deduced that the two vector potentials are directly proportional and linked by the following expression

\[
\Psi^r = \frac{-\rho_f(\omega - \gamma c)^2 \phi K}{\alpha P_j(\omega - \gamma c)^2 K - i(\omega - \gamma c)\phi} \Psi = G(\gamma, \omega) \Psi^*
\]

(23)

The following differential equation is then obtained

\[
\left( \frac{d^2}{dz^2} - \alpha_S^2 \right) \Psi = 0 \quad (24)
\]

with

\[
\alpha_S^2 = \gamma^2 + \zeta^2 - \frac{(\omega - \gamma c)^2}{\mu} \left((1 - \phi)\rho_s + \phi \rho_f \right) \rho_{f G(\gamma, \omega)}
\]

(25)

which yields

\[
\Psi^r = \Psi_S^* e^{-\alpha_S z} = \left\{ \begin{array}{c}
-\psi_{Sx}^* \\
-\psi_{Sy}^* \\
-\psi_{Sz}^*
\end{array} \right\} e^{-\alpha_S z}
\]

(26)

Signs of the real and imaginary parts of \( \alpha_S \) are chosen following similar considerations as given for the compressional waves. Unlike the compressional waves, only one shear wave is obtained. Constants \( \Phi_1^*, \Phi_2^*, \psi_{Sx}^*, \psi_{Sy}^* \) and \( \psi_{Sz}^* \) are determined using the boundary conditions.
3.2 Analytical expressions for transformed displacements

From the Helmholtz decomposition relative to $u$, Eq. (7), written in the wavenumber domain, expressions for the transformed solid displacements are written as

$$
\begin{align*}
\begin{bmatrix}
    u^*_x \\
    u^*_y \\
    u^*_z 
\end{bmatrix} &= \Phi^*_1 e^{\alpha P_1 z} + \Phi^*_2 e^{\alpha P_2 z} + e^{-\alpha S z} \\
    = \begin{bmatrix}
    i\gamma \\
    i\zeta \\
    -\alpha P_1 \\
    -\alpha P_2 \\
    B \\
    C
\end{bmatrix} = [Q_u]
\end{align*}
$$

with $D = \frac{i}{\alpha S} (\gamma B + \zeta C)$. $\Phi^*_1$, $\Phi^*_2$, $B$ and $C$ depend on $\gamma$, $\zeta$ and $\omega$, and are determined by the boundary conditions.

Transformed relative displacements issued from the Helmholtz decomposition, Eq. (7), written in the wavenumber domain are given in a similar way by

$$
\begin{align*}
\begin{bmatrix}
    w^*_x \\
    w^*_y \\
    w^*_z 
\end{bmatrix} &= \begin{bmatrix}
    \Phi^*_1 \\
    \Phi^*_2 \\
    B \\
    C
\end{bmatrix} = [Q_w]
\end{align*}
$$

Fourier transformation of the boundary conditions (10-11) gives

$$
\bar{\sigma}^*_{zz|z=0} = \bar{f}(\gamma, \zeta) = f_0 \frac{\sin(\gamma l_1)}{\gamma l_1} \frac{\sin(\zeta l_2)}{\zeta l_2} \quad (29)
$$

$$
\bar{\sigma}^*_{xz|z=0} = \bar{\sigma}^*_{yz|z=0} = \bar{p}^*_{z=0} = 0 \quad (30)
$$

Then, using relations (3) and (4) with Helmholtz decompositions (7), written in the wavenumber domain and using a matrix notation, the previous integration constants are
obtained from the solution of the following system

\[
[S] \begin{bmatrix}
\Phi_1^* \\
\Phi_2^* \\
B \\
C
\end{bmatrix} = \begin{bmatrix}
\sigma_{xz|z=0}^* \\
\sigma_{yz|z=0}^* \\
\sigma_{zz|z=0}^* \\
\bar{p}_z^*
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
\bar{f}(\gamma, \zeta) \\
0
\end{bmatrix}
\] (31)

with

\[
[S] = \begin{bmatrix}
-2i\mu\gamma \alpha P_1 & -2i\mu\gamma \alpha P_2 & \frac{\mu(\alpha^2 + \gamma^2)}{-\alpha_S} & \frac{\mu\zeta}{-\alpha_S} \\
-2i\mu\zeta \alpha P_1 & -2i\mu\zeta \alpha P_2 & \frac{\mu\zeta}{-\alpha_S} & \frac{\mu(\alpha^2 + \zeta^2)}{-\alpha_S} \\
s_{31} & s_{32} & -2i\mu\gamma & -2i\mu\zeta \\
s_{41} & s_{42} & 0 & 0
\end{bmatrix}
\] (32)

\[
s_{31} = (\alpha^2 P_1 - \gamma^2 - \zeta^2)(\lambda_0 + M\beta^2 + M\beta F_1(\gamma, \zeta, \omega)) + 2\mu_0 P_1
\]

\[
s_{32} = (\alpha^2 P_2 - \gamma^2 - \zeta^2)(\lambda_0 + M\beta^2 + M\beta F_2(\gamma, \zeta, \omega)) + 2\mu_0 P_2
\]

\[
s_{41} = M(-\alpha^2 P_1 + \gamma^2 + \zeta^2)(F_1(\gamma, \zeta, \omega) + \beta)
\]

\[
s_{42} = M(-\alpha^2 P_2 + \gamma^2 + \zeta^2)(F_2(\gamma, \zeta, \omega) + \beta)
\]

Analytical expressions for the vectors of transformed displacements are then given by

\[
\begin{bmatrix}
\bar{u}_x^* \\
\bar{u}_y^* \\
\bar{u}_z^*
\end{bmatrix} = [Q_u] \times [S]^{-1} \begin{bmatrix}
\sigma_{xz|z=0}^* \\
\sigma_{yz|z=0}^* \\
\sigma_{zz|z=0}^* \\
\bar{p}_z^*
\end{bmatrix}
\] (33)

\[
\begin{bmatrix}
\bar{U}_x^* \\
\bar{U}_y^* \\
\bar{U}_z^*
\end{bmatrix} = \left(\frac{1}{\phi}[Q_w] + [Q_u]\right) \times [S]^{-1} \begin{bmatrix}
\sigma_{xz|z=0}^* \\
\sigma_{yz|z=0}^* \\
\sigma_{zz|z=0}^* \\
\bar{p}_z^*
\end{bmatrix}
\] (34)
where \([Q_u]\) and \([Q_w]\) are defined by expressions (27) and (28), \([S]\) by (32). We obtain thus the displacements in the wavenumber domain. Displacements in the real spatial domain are calculated by the use of an inverse Fourier transform performed numerically with an FFT algorithm.

4 Numerical results

4.1 Viscoelastic case

First we propose to scale down the present study to a viscoelastic one-phase medium for which previous results are already available in the bibliography. To reduce the system to the viscoelastic case, we use the following parameters: \(\phi = \beta = 0, \lambda_0 = \lambda_s, M \to \infty, a = 1\) and \(K \to 0\) and characteristic physical parameters are given by: Young modulus \(E_0 = 269\) MPa; Poisson ratio \(\nu = 0.257\); density of solid grains \(\rho_s = 1,550\) kg m\(^{-3}\); damping coefficient \(\eta = 0.1\). Results are compared with those obtained by Lefeuve-Mesgouez et al. (2002). Fig. 2 shows the amplitude of the transformed vertical displacements. For this reduced example, the solid displacements give the same solution as in Lefeuve-Mesgouez (2002).

4.2 Ground and load features

Two kinds of soil are studied: a stiff ground, of rock type, and a partially saturated softer one of sandy-clay type, respectively denoted as soil A and soil B. The soil mechanical parameters were chosen from a literature review and are given in Tabs. 1 (soil A) and 2 (soil B). A harmonic vertical load, of magnitude \(f_0 = 1\) kN and radial frequency \(\omega = 400\) rad/s \((f = 64\) Hz\), acts uniformly over a rectangular area of width \(l_1 = l_2 = 0.3\) m and moves with speed \(c\).

In the Biot theory, it is usual to introduce a characteristic frequency \(f_c\), which governs
the different behaviours of porous media regarding the frequency dependence. It is defined as

\[ f_c = \frac{\phi}{2\pi \rho_f K} \]  

(35)

Study of ratio \( f/f_c \) is useful to characterise the low or high frequency behaviours of the medium. Dispersion and attenuation features need to be known for the interpretation of the ground response. We obtain the following ratio proportional to permeability \( K \) and radial frequency \( \omega \)

\[ \frac{f}{f_c} = \frac{\rho_f K \omega}{\phi} \]

For a given radial frequency, we get high values of \( f/f_c \) for high permeabilities.

Figs. 3 and 4 show the phase velocity and the damping characteristic distance of \( P_1 \), \( S \) and \( P_2 \) waves as a function of \( f/f_c \) for soil B. The phase velocity defined as \( v_\phi = \omega/\Re(k) \), (\( k \) is the complex wavenumber), illustrates the dispersion phenomenon. The damping characteristic distance corresponds to \( \delta = 1/|\Im(k)| \) and gives by means of a characteristic distance the attenuated feature of the different waves.

For low values of \( f/f_c \), all three body waves present some dispersion and attenuation but the compressional wave of the second kind clearly appears to be more attenuated and presents a very low velocity. In such a configuration, its influence is negligible. The viscous coupling is dominant and cancels out the relative motion between solid and fluid displacements. The medium tends toward an equivalent undrained one-phase medium. Low frequency limits for \( P_1 \) and \( S \) phase velocities are due to the average density of the equivalent undrained one-phase soil.

For higher values of \( f/f_c \), dispersive features for \( P_1 - S - P_2 \) waves disappear and all three damping characteristic distances tend to be of similar magnitude. The medium presents a “real” drained two-phase behaviour and the compressional wave of the second
kind can be visualised. The elastic and inertial couplings dominate.

For the case of moving loads, another remark has to be noted. It concerns the existence of Doppler effect. For sub-Rayleigh regimes, considering only the Rayleigh wave, Doppler effect for an observer located on a line defining an angle $\alpha$ with the load direction line, the apparent radial frequency of the load is given by: $\omega' = \omega/(1 - M_R \cos \alpha)$. Thus, the apparent frequencies cover a larger range, yielding higher values of ratio $f/f_c$. The same reasoning can be done for the body waves.

For soil $A$, high frequency limits for the different phase velocities are quickly reached. They are

$$v_{\phi P_1} = 3771 \text{ ms}^{-1}, \quad v_{\phi S} = 2263 \text{ ms}^{-1}, \quad v_{\phi R} = 2063 \text{ ms}^{-1}, \quad v_{\phi P_2} = 1299 \text{ ms}^{-1}$$

where the Rayleigh wave speed is approached by

$$c_R \approx \frac{0.87 + 1.12 \times \nu}{1 + \nu} c_S$$

(36)

These numerical values are in agreement with the results obtained by Coussy (1991) in a more restrictive framework for which there is neither dispersion nor attenuation.

For soil $B$, the wave speeds are given in Tab. 3. For the totally saturated case, values are thus as follows

$$v_{\phi P_1} = 1870 \text{ ms}^{-1}, \quad v_{\phi S} = 530 \text{ ms}^{-1}, \quad v_{\phi R} = 490 \text{ ms}^{-1}, \quad v_{\phi P_2} = 695 \text{ ms}^{-1}$$

In this case, the $P_2$ wave speed value is between the Rayleigh and the $P_1$ wave speeds. Nevertheless, when the ground is partially saturated, the speeds of the compressional waves, both $P_1$ and $P_2$, decrease and the $P_2$ wave speed can be lower than the Rayleigh wave speed. From Tab. 3, we can see that, for a 99.9% partially saturated soil, $c_{P_2} = 280$ ms$^{-1}$ and even 40 ms$^{-1}$ for a 95% partially saturated soil.
4.3 Transformed domain results

Figs. 5, 6 and 7 present results in the transformed domain for soil A, with a high permeability, \( K = 10^{-5} \text{m}^3\text{kg}^{-1}\text{s} \), (low viscous coupling) corresponding to the ratio \( f/f_c = 10 \). Note that in order to ensure visibility of the \( P2 \) wave which differentiates a Biot poroelastic material from an elastic material, and also for a thorough test of the method, the parameter \( K \) is larger than normal values.

For the non-zero damping model, the Rayleigh poles in the \((\gamma, \zeta)\) plane lie on the curves, see Lefèvre-Mesgouez et al. (2000)

\[
\zeta^2 + (1 - M_R^2) \left( \gamma + \frac{k_R M_R}{1 - M_R^2} \right)^2 = \frac{k_R^2}{1 - M_R^2}
\]  
(37)

with the Mach number \( M_R = c/c_R \) related to the Rayleigh wave speed. Then, for \( M_R < 1 \) (sub-Rayleigh regime), the Rayleigh poles lie on an ellipse and for \( M_R > 1 \) (super-Rayleigh regime), the poles are situated on a hyperbola. For the branch points, a similar analysis is possible replacing \((k_R, M_R)\) with \((k_{P1}, M_{P1})\), \((k_{P2}, M_{P2})\), or \((k_S, M_S)\), where \( M_{P1} = c/c_{P1} \), \( M_{P2} = c/c_{P2} \) and \( M_S = c/c_S \). Fig. 5 compares the location of the dominant peaks obtained using the modified hysteretic damping model and the theoretical location of the Rayleigh poles for zero damping, for the transformed vertical fluid surface displacement. Also, the maxima due to the \( P1 \) and \( P2 \) waves can be seen and compared with the theoretical ellipses / hyperbolas. Results show very good agreement in both sub-Rayleigh and super-Rayleigh regimes. No peak due to the shear wave can be seen here because, for the low viscous coupling chosen, the shear wave does not exist in the fluid phase.

The real part of the transformed vertical fluid surface displacements along the \( \gamma - \) axis is plotted in Fig. 6 for different wave speeds. Peaks due to the Rayleigh wave give a negative contribution whereas peaks due to the \( P1 \) and \( P2 \) waves are positive. As the load speed tends towards the Rayleigh wave speed, the peaks in the negative wavenumber
domain decrease in height and move towards $-\infty$. This suggests that the contribution of these peaks to waves propagating in front of the load are less and less significant. On the other hand, the peaks in the positive wavenumbers move towards 0. When the wave speed passes through the Rayleigh wave speed, the peak in the negative wavenumbers jumps to the positive wavenumbers and moves from $+\infty$ towards 0: it reflects the move from the ellipse to the hyperbola due to the change in the speed regime as seen in Fig. 5. Thus, for super-Rayleigh regimes, the displacements along the $x-$ axis are formed by the superposition of two waves propagating in the negative $x-$direction at speeds $c+c_R$ and $c-c_R$ respectively, plus the contribution of body waves.

Fig. 7 presents the same analysis for the transformed vertical solid surface displacements along the $\gamma-$ axis. The main conclusions are similar to the ones obtained for the fluid phase except that the amplitudes of displacements are higher and that the shear wave is clearly seen whereas the influence of the $P2$ wave is not. Fluid and solid phases have thus different behaviours in agreement with the high ratio $f/f_c$.

When changing the value of permeability to a lower one, the ratio $f/f_c$ decreases, thus the solid and fluid displacements over the surface of the ground are closer to each other because of the stronger coupling: the porous ground tends towards a one-phase medium.

4.4 Vibration transmission solutions

4.4.1 Inverse Fast Fourier transform

For non zero damping, the inverse Fourier transform can be calculated numerically. The solutions have been obtained by the well-known Fast Fourier transform (FFT) algorithm, see Brigham (1974) for instance. To compute the inverse transform accurately with a discrete transform, the integrals must be truncated at sufficiently high values to avoid
distortion of the results by aliasing, while the mesh of calculated functions must be fine enough to clearly represent the details of the functions seen in the transformed domain. Consequently, the limit values for wavenumbers were chosen much higher than the ellipse / hyperbola location discussed in the previous section: the Rayleigh wavenumber is calculated for each case and the grid is taken up to $40 \times k_R$. With the modified hysteretic damping, Rayleigh poles and branch points have complex values, which avoids singularities. It was found that, with the properties used here, an FFT with $2048 \times 2048$ points, a range of $|\gamma|, |\zeta| \leq 8.2 \text{ m}^{-1}$ and $|\gamma|, |\zeta| \leq 30.8 \text{ m}^{-1}$, and a wavenumber step given as $d\gamma = d\zeta \simeq 0.8 \times 10^{-2} \text{ m}^{-1}$ and $d\gamma = d\zeta \simeq 3 \times 10^{-2} \text{ m}^{-1}$ for soil A and soil B respectively, satisfied both these requirements. The modified hysteretic damping parameter is respectively $\eta = 0.01$ and $\eta = 0.1$, see Tabs. 1 and 2.

4.4.2 Influence of the load speed for the stiff ground

In this section, Figs. 8 and 9 present results in the spatial domain for the set of parameters presented in section 4.3 with high ratio $f/f_c = 10$ to emphasize the $P2$ wave as previously stated.

Fig. 8 presents the contour lines of the modulus of the vertical fluid surface displacements for a sub-Rayleigh speed, $M_R = 0.5$, and a super-Rayleigh speed, $M_R = 1.5$. The figure has a contour of maximum values $4.43 \times 10^{-9}$ m and $1.21 \times 10^{-8}$ m for the sub-Rayleigh and the super-Rayleigh cases respectively. Contour lines are plotted with isoline step-values $2.9 \times 10^{-10}$ for both cases, which gives 18 and 40 contour lines respectively. For a low Mach number, the distribution of displacements is almost symmetrical and is localised in the near field. As the load speed increases and passes through the Rayleigh wave speed, a pronounced change in the displacements occurs: the distribution of displacements spreads behind the load and is contained between two Mach-lines of higher
displacements delimiting two regions of displacements. The displacement is almost equal to zero in front of the load. Moreover, two other Mach-lines of less importance can be seen in this case due to the $P_2$ wave. As the Rayleigh wave is dominant over the surface of the ground, the $P_2$ Mach-lines are very low. A plot of the vertical fluid displacements in depth allows a better visualisation of the $P_2$ Mach cone, Fig. 9.

Figure 10 shows the solid and fluid vertical displacements for lower values of permeability which are more realistic for stiff grounds (ranging from $10^{-5}$ to $10^{-9}$ m$^3$kg$^{-1}$s$^{-f/f_c}$ from 10 to 0.001). For the three cases, solid vertical displacements are nearly identical and only the fluid phase is affected. For the lower permeability, the ground acts as a one-phase medium since the solid and fluid displacements are equal to each other. In this case, with appropriate elastic parameters, a one-phase viscoelastic approach can be satisfactory to describe the ground behaviour. Nevertheless, with the macroscopic two-phase description of the ground, the influence of parameters such as fluid compressibility or permeability can be studied, that is not the case any more with viscoelasticity approach: Theodorakopoulos (2003) shows for instance that the influence of fluid compressibility has important consequences on the ground response, for soft grounds. We propose to analyse this aspect in the next section.

### 4.4.3 Partially saturated softer ground

As previously stated, the wave velocities of soil $B$ are lower involving the existence of higher displacements along Mach cones for accessible values of the load speed. Moreover, for partially saturated soils, the speeds of compressional waves can drastically change. For instance, for the previous characteristics, the $P_2$ wave velocity can be divided by 2.5 with only a saturation of 99.9%, see Tab. 3. This observation has thus a consequence on the existence of the corresponding Mach cone which then occurs for lower values of load
speed.

In the transformed domain, a similar situation as the one obtained for soil A occurs with peaks due to each wave located on hyperbolas for supersonic regimes. Fig. 11 presents results in the spatial domain for $M_R = 2.78$, and still $f/f_c = 10$ to get a contribution of the $P2$ wave. It shows the Mach cones due to the $P2$, $R$ and $P1$ waves for soil B’s characteristics with a 99.9% saturation. In this case, speed values are given in Tab. 3 and each of the waves’ critical regimes are surpassed. 17 contour lines are plotted with isocline step-values $7.8 \times 10^{-9}$ and the maximum value equals $1.34 \times 10^{-7}$ m. We note that the displacements are higher than the ones obtained for soil A. Fig. 12 presents a cut along the line $x = -3.9$ m of the real domain and the vertical solid displacements are added to the figure. We can see that the $P2$ wave can have a contribution of similar amplitude compared to the others in the fluid phase.

Fig. 13 shows the vertical solid displacements for a more typical value of the permeability ($K = 10^{-9}$ m$^2$kg$^{-1}$s - $f/f_c = 0.001$) and for a sub-Rayleigh regime ($M_R = 0.25$). Even for this low value of $f/f_c$, several parameters of the Biot theory can influence the solid behaviour. For instance, when studying the influence of saturation (linked to the first Biot coefficient) on the solid displacements, even for the sub-Rayleigh regime, a significant change in the amplitudes has to be noted: Fig. 13 shows that the maximum of solid displacements occurs under the load and exhibits a 20% difference between the saturated case and the 95% partially saturated case. As the saturation is connected with the fluid compressibility (see Eq. (14)), this result is similar to the conclusion obtained by Theodorakopoulos for a softer medium.

In order to get more accessible values of the body wave speeds, a partially saturated softer ground is studied in the following. Young modulus and Poisson ratio are given by: $E = 1.2 \times 10^8$ Pa and $\nu = 0.3$ which yields the shear wave speed $c_S = 154$ m.s$^{-1}$. The first
Biot coefficient is given the value: \( M = 0.275 \times 10^8 \) Pa (i.e. \( S_R = 97\% \)) which leads to \( c_{P1} = 311 \, \text{m.s}^{-1} \) (low frequency limits). Note that only Young modulus and the first Biot coefficient have been modified in order to get lower values of body wave speeds. The value of permeability is: \( K = 10^{-7} \, \text{m}^3\text{kg}^{-1}\text{s}^{-f/f_c} = 0.1 \). Figs. 14 (a) and (b) examine the maximum surface vertical solid displacement versus the load speed for different values of radial frequency, for the poroviscoelastic model and its associated equivalent viscoelastic model. The equivalent one-phase model has been obtained by calculating Lamé constants from the values of body wave speeds and equivalent density (\( \rho_{eq} = 1960 \, \text{kg.m}^{-3} \)). The displacements are normalised with respect to the displacement response calculated for the zero load speed and the velocity is normalised with respect to the shear wave speed. First, it can be seen that the maximum vertical displacement increases with the load speed. For low load speeds the increase is low. But when it approaches the Rayleigh wave speed, the maximum displacement can be up to 40\% higher than the values obtained for the “static” case (\( c/c_S = 0 \)). The value of the radial frequency affects the peak around the Rayleigh speed: it is magnified for lower radial frequency. If we compare the equivalent one-phase model to the two-phase model, for low load velocities, similar results are obtained. Differences appear for higher load speeds: the equivalent viscoelastic model accentuates the maximum vertical displacement. For the middle radial frequency (200 rad.s\(^{-1}\) - 32 Hz) presented in Fig. 14(b), the difference is up to 8\%. In conclusion, the equivalent viscoelastic model overestimates the maximum displacement beyond \( c/c_S = 0.5 \).

5 Conclusion

A three-dimensional semi-analytical model, to study fluid and solid displacements in wavenumber and spatial domains, has been presented for the case of a high-speed moving
load over a totally or partially saturated poroviscoelastic half-space. Useful analysis in the transformed domain allows a better understanding of the influence of the different waves yielding, for instance, the visualisation of the contribution of the compressional wave of the second kind. Moreover, with such an analysis, we emphasize the different regimes that exist when the load speed passes through critical values (Rayleigh wave speed, compressional and shear wave speeds): maximum values of transformed displacements lie on ellipses or hyperbolas depending on the load speed. The analysis in the spatial domain has focused specifically on the fluid phase behaviour, with the visualisation of Mach cones for super-critical regimes. Results have also shown that solid displacements are affected by the effect of the saturation of the medium. Moreover, it is interesting to note that such theoretical approaches can also be used as benchmarks to validate numerical simulations.

References


Smeulders, D.M.J., De La Rosette, J.P.M., Van Dongen, M.E.H., 1992. Waves in par-
Figures Captions

Fig. 1. Three-dimensional geometry for the rectangular harmonic moving load problem.

Fig. 2. Amplitude of transformed vertical solid surface displacements along the $\gamma-$ axis for the degenerated viscoelastic model.

Fig. 3. Phase velocity as a function of $f/f_c$ for $P_1$ (solid line), $S$ (dashed line) and $P_2$ (dotted line) waves.

Fig. 4. Damping characteristic distance $\delta$ as a function of $f/f_c$ for $P_1$ (solid line), $S$ (dashed line) and $P_2$ (dotted line) waves.

Fig. 5. Comparison between theoretical Rayleigh poles with no damping and contour levels of the transformed vertical fluid surface displacements with modified hysteretic damping: theoretical Rayleigh poles in white dashed line, theoretical $P_2$ branch points in black dashed - dotted line and theoretical $P_1$ branch points in black dashed line; (a) sub-Rayleigh wave speed case, $M_R = 0.5$, (b) super-Rayleigh wave speed case, $M_R = 1.5$.

Fig. 6. Evolution of the real part of transformed vertical fluid surface displacements in the wavenumber domain with increasing load speed, along the line $\zeta = 0$: (a) $M_R = 0$, (b) $M_R = 0.5$, (c) $M_R = 1$, (d) $M_R = 1.5$.

Fig. 7. Evolution of the real part of transformed vertical solid surface displacements in the wavenumber domain with increasing load speed, along the line $\zeta = 0$: (a) $M_R = 0$, (b) $M_R = 0.5$, (c) $M_R = 1$, (d) $M_R = 1.5$.

Fig. 8. Contour lines of the vertical fluid surface displacements (a) sub-Rayleigh wave speed case, $M_R = 0.5$ and (b) super-Rayleigh wave speed case, $M_R = 1.5$.

Fig. 9. Contour lines of the vertical fluid displacements in depth for a super-Rayleigh wave speed case, $M_R = 1.5$.

Fig. 10. Vertical solid (solid line) and fluid (dotted line) displacements for different permeability values: (a) $K = 10^{-5}$ m$^3$kg$^{-1}$s - $f/f_c = 10$, (b) $K = 10^{-7}$ m$^3$kg$^{-1}$s - $f/f_c = 0.1$, ...
(c) \( K = 10^{-9} \text{m}^3\text{kg}^{-1}\text{s} - f/f_c = 0.001 \).

Fig. 11. Visualisation of the Mach cones relative to the \( R, P2 \) and \( P1 \) waves for soil \( B \) in the super-critical regime \( M_R = 2.78 \): (a) 3D visualisation, (b) contour lines.

Fig. 12. Contribution of the \( R, P1 \) and \( P2 \) waves along the line \( x = -3.9 \text{ m} \) in the spatial domain for the vertical solid (solid line) and fluid (dashed line) surface displacements.

Fig. 13. Influence of the saturation on the vertical solid displacements for \( K = 10^{-9} \text{m}^3\text{kg}^{-1}\text{s} - f/f_c = 0.001 \) and \( M_R = 0.25 \): totally saturated case (solid line), 99.9\% partially saturated case (dashed line), 95\% partially saturated case (dotted line).

Fig. 14. Influence of the load speed on the normalised maximum vertical solid displacement for the two-phase model (no symbol) and the equivalent one-phase model (cross symbols) for different radial frequencies: (a) \( \omega = 400 \text{ rad.s}^{-1}, f = 64 \text{ Hz} \) -dotted lines-, \( \omega = 100 \text{ rad.s}^{-1}, f = 16 \text{ Hz} \) -dashed lines-, (b) \( \omega = 200 \text{ rad.s}^{-1}, f = 32 \text{ Hz} \) -solid lines-. 
Table Captions

Tab. 1. Soil characteristics for soil A (stiff ground).

Tab. 2. Soil characteristics for soil B (softer ground).

Tab. 3. Changes due to partial saturation for soil B.
<table>
<thead>
<tr>
<th>Young Modulus of drained porous media $E_0$ (MPa)</th>
<th>Poisson Ratio $\nu$</th>
<th>Bulk Modulus of solid grains $K_s$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 000</td>
<td>0.2</td>
<td>40 000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bulk Modulus of fluid component $K_f$ (MPa)</th>
<th>First Biot coefficient $M$ (MPa)</th>
<th>Second Biot coefficient $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 200</td>
<td>5 270</td>
<td>0.72</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Density of solid grains $\rho_s$ (kg m$^{-3}$)</th>
<th>Density of fluid component $\rho_f$ (kg m$^{-3}$)</th>
<th>Hydraulic permeability coefficient $K$ (m$^3$kg$^{-1}$s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2600</td>
<td>1000</td>
<td>10e-9-10e-5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Porosity $\phi$</th>
<th>Tortuosity coefficient $a$</th>
<th>Damping coefficient $\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>1.2</td>
<td>0.01</td>
</tr>
<tr>
<td>property</td>
<td>value</td>
<td>property</td>
</tr>
<tr>
<td>--------------------------------</td>
<td>----------------</td>
<td>--------------------------------</td>
</tr>
<tr>
<td>Young Modulus of drained porous media $E_0$ (MPa)</td>
<td>1 200</td>
<td>Poisson Ratio $\nu$</td>
</tr>
<tr>
<td>Bulk Modulus of fluid component $K_f$ (MPa)</td>
<td>2 200</td>
<td>First Biot coefficient $M$ (MPa)</td>
</tr>
<tr>
<td>Density of solid grains $\rho_s$ (kg m$^{-3}$)</td>
<td>2600</td>
<td>Density of fluid component $\rho_f$ (kg m$^{-3}$)</td>
</tr>
<tr>
<td>Porosity $\phi$</td>
<td>0.4</td>
<td>Tortuosity coefficient $a$</td>
</tr>
<tr>
<td>Saturation $S_R$ (%)</td>
<td>100</td>
<td>99.99</td>
</tr>
<tr>
<td>----------------------</td>
<td>------</td>
<td>-------</td>
</tr>
<tr>
<td>First Biot coefficient</td>
<td>5100</td>
<td>1680</td>
</tr>
<tr>
<td>$S$ wave speed (ms$^{-1}$)</td>
<td>530</td>
<td>530</td>
</tr>
<tr>
<td>$P_1$ wave speed (ms$^{-1}$)</td>
<td>1870</td>
<td>1285</td>
</tr>
<tr>
<td>$P_2$ wave speed (ms$^{-1}$)</td>
<td>695</td>
<td>585</td>
</tr>
</tbody>
</table>
Poroviscoelastic halfspace

\[ \sigma_{zz}|_{z=0} = f(x,y)e^{i\omega t} \]

\( x_1, x_2, x_3 \) fixed frame of reference
\( x, y, z \) moving frame of reference
Figure 11

(a)

(b)
Figure 14

(a)

(b)